BASIC CONCEPTS IN LAPLACE TRANSFORMS

Definition 1: Laplace Transform of f(t)

If f(t) is a function of t defined for all $t \geq 0$, then the Laplace transform of f(t) denoted by $L[f(t)]$ is defined as

$$L[f(t)] = F(s) = \int_0^\infty e^{-st} f(t)\,dt, \text{ where } s>0, \text{ provided the integral exist}$$

Definition 2: Condition for the Existence of Laplace Transform

The laplace transforms of the function f(t) defined for $t \geq 0$ exists if f(t) is

(i) Piecewise continuous in every finite interval in the range $t \geq 0$.
(ii) Of the exponential Order

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Definition 3: Inverse Laplace transforms

Inverse Laplace transform of F(s), denoted by $L^{-1}[F(s)]$, is defined as $L^{-1}[F(s)] = f(t)$, where $L[f(t)] = F(s)$

Definition 4: Condition for the Existence of Inverse Laplace Transform

(i) $\lim_{s \to \infty} F(s) = 0$ (ii) $\lim_{s \to \infty} s \cdot F(s)$ is finite .

Definition 5: Exponential Order

A function f(t) is said to be of exponential order if

$\lim_{t \to \infty} e^{-st} f(t)$ is finite

Definition 6: Periodic Functions

A function f(t) is said to be Periodic if there exists a constant P (>0) such that $f(t+P) = f(t)$, for all t

Definition 7: Laplace Transform of Periodic Functions

If f(t) is a periodic function then $L[f(t)] = \dfrac{1}{(1-e^{-Ps})} \int_0^P e^{-st} f(t)\,dt$

STANDARD RESULTS IN LAPLACE & INVERSE TRANSFORMS

Sl.No.	LAPLACE TRANSFORMS	INVERSE LAPLACE TRANSFORMS
1	$L[1] = \frac{1}{s}$	$L^{-1}\left[\frac{1}{s}\right] = 1$
2	$L[e^{at}] = \frac{1}{s-a}$	$L^{-1}\left[\frac{1}{s-a}\right] = e^{at}$
3	$L[e^{-at}] = \frac{1}{s+a}$	$L^{-1}[\frac{1}{s-a}] = e^{at}$
4	$L[t] = \frac{1}{s^2}$	$L^{-1}\left[\frac{1}{s^2}\right] = t^2$
5	$L[t^2] = \frac{2}{s^3}$	$L^{-1}\left[\frac{1}{s^3}\right] = \frac{t^2}{2}$
6	$L[t^n] = \frac{n!}{s^{n+1}}$ **,** n is a + integer	$L^{-1}\left[\frac{1}{s^n}\right] = \frac{t^{n-1}}{(n-1)!}$
7	$L[\sin at] = \frac{a}{s^2+a^2}$	$L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$
8	$L[\text{Cos } at] = \frac{s}{s^2+a^2}$	$L^{-1}\left[\frac{s}{s^2+a^2}\right] = \text{Cos } at$
9	$L[\text{Sinh}(at)] = \frac{a}{s^2-a^2}$	$L^{-1}\left[\frac{1}{s^2-a^2}\right] = \frac{1}{a}\text{Sinh}(at)$
10	$L[\text{Cosh}(at)] = \frac{s}{s^2-a^2}$	$L^{-1}\left[\frac{s}{s^2-a^2}\right] = \text{Cosh}(at)$
11	$U_a(t) = \begin{cases} 0, \text{ when } t<a \\ 1, \text{ when } t>a, \text{where } a\ge 0 \end{cases}$ $U_a(t)$ is called unit – step function **L[** $U_a(t)$ **] =** $\frac{e^{-as}}{s}$	$\mathbf{L}^{-1}\left[\frac{e^{-as}}{s}\right] = U_a(t)$
12	$\delta_a(t) = \begin{cases} \frac{1}{h}, \text{ when } a-\frac{h}{2}<t<a+\frac{h}{2} \\ 0, \text{ otherwise} \end{cases}$ $\delta_a(t)$ is called Dirac - Delta function $L[\delta_a(t)] = e^{-as}$ and $L[\delta(t)] = 1$ where $a = 0$	$L^{-1}[1] = \delta(t)$

1. Problems on Laplace Transform of Standard Functions

1. Linearity Property of Laplace Transform

1. $L[k_1 f_1(t) \pm k_2 f_2(t)] = k_1 L[f_1(t)] \pm k_2 L[f_2(t)]$

2. $L^{-1}[k_1 F_1(s) \pm k_2 F_2(s)] = k_1 L^{-1}[F_1(s)] \pm k_2 L^{-1}[F_2(s)]$

(A) Find the Laplace Transforms for the following function

Class Work Problems		
(i) $a + e^{-bt}$	(v) $\sin^2 at$	(ix) $\sin at \ \cos bt$
(ii) $1 + e^{bt} - 5\sin 3t$	(vi) $\cos^3 at$	(x) $\sin 2t \ \cos 3t$
(iii) $a - e^{-bt} - c\cos kt$	(vii) $\sin^2 3t$	(xi) $(1+t)^2$
(iv) $\sin 4t - 4\sinh 2t$	(viii) $\cos^3 2t$	(xii) $a\sqrt{t} + \frac{b}{\sqrt{t}} + ct^{\frac{3}{2}}$

Practice Problems	Answers of Practice Problems
1 $\cos(\alpha t + \beta)$	$\cos\beta \frac{s}{s^2+\alpha^2} - \sin\beta \frac{\alpha}{s^2+\alpha^2}$
2. $\cos^2 3t + \sin^3 4t$	$\frac{1}{2}\left(\frac{1}{s} + \frac{s}{s^2+36}\right) + \frac{3}{s^2+16} - \frac{3}{s^2+144}$
3. $(\sqrt{t} - 1)^2$	$\frac{1}{s^2} - \frac{\sqrt{\pi}}{s^{\frac{3}{2}}} + \frac{1}{s}$
4. e^{3t+5}	$e^5\left(\frac{1}{s-3}\right)$
5. $\cosh 10t - \sinh \pi t$	$\frac{s}{s^2-100} - \frac{\pi}{s^2-\pi^2}$
6. $\cos 2t \ \cos 3t$	$\frac{1}{2}\left(\frac{s}{s^2+25} + \frac{s}{s^2+1}\right)$
7. $2 + e^{-3t}$	$\frac{2}{s} + \frac{1}{s+3}$

PROPERTIES OF LAPLACE TRANSFORMS

2. First Shifting theorem

If $L[f(t)] = F(s)$ then

(i) $L\left[e^{at}[f(t)]\right] = F(s-a)$ (ii) $L\left[e^{-at}[f(t)]\right] = F(s+a)$

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3. If $L[f(t)] = F(s)$ then

(i) $L[t\,[f(t)]] = -\frac{d}{ds}[F(s)]$ (ii) $L[t^2\,[f(t)]] = (-1)^2 \frac{d^2}{ds^2}[F(s)]$

(iii) $L\left[t^3\,[f(t)]\right] = (-1)^3 \frac{d^3}{ds^3}[F(s)]$ (iv) In General, $L\left[t^n\,[f(t)]\right] = (-1)^n \frac{d^n}{ds^n}[F(s)]$

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4. If $L[f(t)] = F(s)$ and $\lim\limits_{t \to 0} \frac{f(t)}{t}$ exists finite, then

(i) $L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)\,ds$ (ii) $L\left[\frac{f(t)}{t^2}\right] = \int_s^\infty \int_s^\infty F(s)\,ds^2$

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5. Laplace Transforms of Derivatives

If $L[f(t)] = F(s)$ then

(i) $L\left[f'(t)\right] = s\,L[f(t)] - f(0)$ (ii) $L\left[f''(t)\right] = s^2\,L[f(t)] - s\,f(0) - f'(0)$

In General (iii) $L\left[f^n(t)\right] = s^n\,L[f(t)] - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots\ldots - f^{n-1}(0)$

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6. Laplace Transforms of Integrals

If $L[f(t)] = F(s)$ then (i) $L\left[\int_0^t f(t)\,dt\right] = \frac{1}{s} L[f(t)]$ (ii) $L\left[\int_0^t \int_0^t f(t)\,dt^2\right] = \frac{1}{s^2} L[f(t)]$

Problems on Properties of Laplace Transforms

(B) Find the Laplace Transforms for the following function

Class Work Problems		
1. $e^{at}\cos bt$	10. $t\,e^{-2t}\sin 3t$	19. $\frac{\sin 3t\cos 2t}{t}$
2. $e^{-2t}\sin 5t$	11. $t\;e^{3t}\;\cos 4t$	20. $\frac{\cos at}{t}$
3. $e^{3t}\cosh t$	12. $t\,e^{3t}\;\sin 2t\sin 3t$	21. $\int_0^t e^{2t}dt$
4. $e^{-3t}\sin^2 4t$	13. $\frac{1-e^{2t}}{t}$	22. $\int_0^t e^{-t}\sinh 2t\,dt$
5. $e^{4t}\;\cos 5t\sin 2t$	14. $\frac{1-\cos at}{t}$	23. $\int_0^t t\sin^2 t\,dt$
6. te^{at}	15. $\frac{e^{-at}-e^{-bt}}{t}$	24. $e^{-t}\int_0^t t\cos t\,dt$
7. $t\sin 3t$	16. $\frac{e^{at}-\cos bt}{t}$	25. $e^{-t}\int_0^t \frac{\sin at}{t}\,dt$
8. $t\cos^2 2t$	17. $\frac{\cos at-\cos bt}{t}$	26. $\int_0^t te^{3t}\cosh 4t\,dt$
9. $t^2\cosh 4t$	18. $\frac{\sin^2 t}{t}$	27. $e^{4t}\int_0^t \frac{\sin 3t\cos 2t}{t}\,dt$

Practice Problems

1. $\dfrac{t^2}{e^{3t}}$ Ans : $\dfrac{2}{(s+3)^3}$

2. $e^{2t}(1+t)^2$ Ans : $\dfrac{1}{s-2}+\dfrac{2}{(s-2)^2}+\dfrac{2}{(s-2)^3}$

3. $e^{-2t}\sin^3 4t$ Ans : $\dfrac{3}{(s+2)^2+16}-\dfrac{3}{(s+2)^2+144}$

4. $t\cos^3 2t$ Ans : $-\dfrac{1}{4}\left[\dfrac{12-3s^2}{\left(s^2+4\right)^2}+\dfrac{36-s^2}{\left(s^2+36\right)^2}\right]$

5. $t^2 \sinh 5t$ Ans : $\dfrac{10\left(3s^2+25\right)}{\left(s^2-25\right)^3}$

6. $t\, e^{-4t}\cosh 4t$ Ans : $\dfrac{(s+4)^2+16}{\left((s+4)^2-16\right)^2}$

7. $\dfrac{e^{2t}-\cos 3t}{t}$ Ans : $\log\left[\dfrac{\left(s^2+9\right)^{\frac{1}{2}}}{s-2}\right]$

8. $\dfrac{e^{-t}-e^{-2t}}{t}$ Ans : $\log\left(\dfrac{s+2}{s+1}\right)$

9. $\dfrac{1-\cos t}{t}$ Ans : $\log\left[\dfrac{\left(s^2+1\right)^{\frac{1}{2}}}{s}\right]$

10. $\dfrac{\cos at-\cos bt}{t}$ Ans : $\log\left[\dfrac{\left(s^2+b^2\right)^{\frac{1}{2}}}{\left(s^2+a^2\right)^{\frac{1}{2}}}\right]$

11. $\dfrac{e^{at}}{t}$ Ans : Laplace Transform doesnot exists

12. $e^{t}\int_{0}^{t}\frac{\sin^{2}t}{t}dt$ Ans : $\frac{1}{2(s-1)}\log\left[\frac{\left((s-1)^{2}+4\right)^{\frac{1}{2}}}{s-1}\right]$

13. $\int_{0}^{t}e^{4t}t^{2}\sinh 3t\,dt$ Ans : $\frac{6}{s}\left[\frac{3(s-4)^{2}+9}{\left((s-4)^{2}-9\right)^{2}}\right]$

Laplace Transform of periodic functions

If f(t) is a periodic function with period P so that $f(t+P)=f(t)$ then

$$L[f(t)]=\frac{1}{1-e^{-Ps}}\int_{0}^{P}e^{-st}\,f(t)\,dt$$

Find the Laplace transform of the following functions

1. $f(t)=\begin{cases}1, & 0<t<a\\ -1, & a<t<2a\end{cases}$ given that $f(t+2a)=f(t)$

2. $f(t)=\begin{cases}t, & 0<t<a\\ 2a-t, & a<t<2a\end{cases}$ given that $f(t+2a)=f(t)$

3. $f(t)=\begin{cases}t, & 0<t<\frac{\pi}{2}\\ \pi-t, & \frac{\pi}{2}<t<\pi\end{cases}$ given that $f(t+\pi)=f(t)$

4. $f(t)=\begin{cases}E\sin wt, & 0<t<\frac{\pi}{\omega}\\ 0, & \frac{\pi}{\omega}<t<\frac{2\pi}{\omega}\end{cases}$ given that $f\left(t+\frac{2\pi}{\omega}\right)=f(t)$

Practice Problems

Find the Laplace transform of the following functions

1. $f(t) = \begin{cases} t, & 0 < t < 1 \\ 2 - t, & 1 < t < 2 \end{cases}$ given that $f(t+2) = f(t)$

Ans: $\frac{1}{s^2}\tanh\frac{s}{2}$

2. $f(t) = \begin{cases} E, & 0 < t < \frac{a}{2} \\ -E, & \frac{a}{2} < t < a \end{cases}$ given that $f(t+a) = f(t)$

Ans : $\frac{E}{s}\tanh\frac{as}{4}$

3. $f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$ given that $f(t+2\pi) = f(t)$

Ans: $\frac{1}{s^2+1}\left(\frac{1+e^{-\pi s}}{1-e^{-\pi s}}\right)$

Initial value Theorem

If $L[f(t)] = F(s)$ then $\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)$

Final value Theorem

If $L[f(t)] = F(s)$ then $\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$

Verify Initial value and Final value theorems for the following functions

1. $f(t) = 1 - e^{-at}$ 2. $f(t) = e^{-2t}\cos 3t$ 3. $f(t) = 1 + e^{-t}(\sin t + \cos t)$

Practice Questions

Verify Initial value and Final value theorems

1. $f(t) = t^2 e^{-3t}$

2. $f(t) = 1 + e^{-t}\sin 3t$

Change of scale Property

If $L[f(t)] = F(s)$ then $L[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$

Problems

1. Given that $L[t\cos t] = \frac{s^2 - 1}{\left(s^2 + 1\right)^2}$, Find (i) $L[t\cos at]$, (ii) $L\left[t\cos\frac{t}{2}\right]$

2. Assuming $L[\sin t]$, Find $L\left[\sin\left(\frac{t}{2}\right)\right]$

Second Shifting Theorem

If $L[f(t)] = F(s)$ and $G(t) = \begin{cases} f(t-a) & \text{for } t > a \\ 0 & \text{for } t \le a \end{cases}$ then $L[G(t)] = e^{-as}F(s)$

Another Form of Second Shifting Theorem

If $L[f(t)] = F(s)$ and $a > 0$ then $L[f(t-a)U(t-a)] = e^{-as}F(s)$

Using Second shifting theorem find the Laplace Transform

1. $G(t) = \begin{cases} \sin\left(t - \frac{\pi}{3}\right) & \text{for } t > \frac{\pi}{3} \\ 0 & \text{for } t \le \frac{\pi}{3} \end{cases}$

2. $G(t) = \begin{cases} (t-2)^3 & \text{for } t > 2 \\ 0 & \text{for } t \le 2 \end{cases}$

Miscellaneous Problems

1. Find the laplace transform of $f(t) = \begin{cases} \sin t & \text{for } 0 < t < \pi \\ 0 & \text{for } t \ge \pi \end{cases}$

2. Using Laplace Transform evaluate $\int_0^\infty e^{-2t}\sin 3t\, dt$

Problems for Practice

1. Find the laplace transform of $f(t)=\begin{cases} e^{t} & \text{for } 0<t<1 \\ 0 & \text{for } t\geq 1 \end{cases}$

Ans : $\dfrac{1-e^{-(s-1)}}{s-1}$

2. Using Laplace Transform evaluate $\int_{0}^{\infty} te^{-3t}\sin 2t\,dt$

Ans: $\dfrac{12}{169}$

INVERSE LAPLACE TRANSFORMS

Class Work Problems

1. $\dfrac{1}{s}+\dfrac{3s}{s^{2}+4}-\dfrac{2}{s^{2}}+\dfrac{5}{s-12}$

2. $\dfrac{2}{s^{2}-9}-\dfrac{4}{s^{6}}$

3. $\dfrac{5s}{s^{2}+10}$

Practice Problems

1. $\dfrac{3}{s^{2}-3}+\dfrac{2}{s^{5}}-\dfrac{7}{s}$ **Ans:** $\sqrt{3}\sinh\sqrt{3}t+\dfrac{t^{4}}{12}-7$

2. $\dfrac{3s}{s^{2}+25}+\dfrac{11}{s+4}$ **Ans:** $3\cos 5t+11e^{-4t}$

Properties of Inverse Laplace Transforms

Property 1: First Shifting Property

If $L[f(t)]=F(s)$ then

(i) $L^{-1}[F(s+a)]=e^{-at}L^{-1}[F(s)]$ (ii) $L^{-1}[F(s-a)]=e^{at}L^{-1}[F(s)]$

Property 2:

$$L^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t L^{-1}[F(s)]\, dt$$

Similarly $L^{-1}\left[\frac{1}{s^2}F(s)\right] = \int_0^t \int_0^t L^{-1}[F(s)]\, dt$

Property 3:

$$L^{-1}[s\,F(s)] = \frac{d}{dt}L^{-1}[F(s)],$$

similarly $L^{-1}\left[s^2\,F(s)\right] = \frac{d^2}{dt^2}L^{-1}[F(s)]$

Property 4:

$$L^{-1}[F(s)] = -t\,L^{-1}[F(s)]$$

Property 5: Second Shifting Property

$$L^{-1}\left[e^{-as}F(s)\right] = \left[L^{-1}[F(s)]\right]_{t \to t-a} U(t-a)$$

(or) $L^{-1}\left[e^{-as}F(s)\right] = f(t-a)\ U(t-a)$

Problems based on Properties of Inverse Laplace Transforms

Find the Inverse Laplace Transforms for the following functions

(1) $\frac{1}{(s-2)^2}$	(2) $\frac{3}{(s+4)^2}$	(3) $\frac{2}{(s+1)^2+49}$
(4) $\frac{s+2}{(s+2)^2+25}$	(5) $\frac{4}{(s-1)^2-4}+\frac{3s}{(s+4)^2-9}$	(6) $\frac{s+2}{s^2-6s+25}$
(7) $\frac{3s-4}{s^2-8s+65}$	(8) $\frac{1}{s(s+1)}$	(9) $\frac{1}{s(s^2+6s+13)}$
(10) $\frac{s+2}{(s^2+4s+5)^2}$	(11) $\tan^{-1}\left(\frac{s+1}{3}\right)$	(12) $\tan^{-1}\left(\frac{a}{s}\right)+\cot^{-1}\left(\frac{s}{b}\right)$
(13) $\log\left(\frac{s+a}{s+b}\right)$	(14) $\log\left(1+\frac{a^2}{s^2}\right)$	(15) $\log\left[\frac{s(s^2+a^2)}{(s^2+b^2)}\right]$
(16) $\frac{e^{-2s}}{(s+1)^3}$	(17) $\frac{s}{a^2s^2+b^2}$	

Practice Problems for Inverse Laplace Transforms based on Properties

1. $\frac{3}{(s+3)^2}$ Answer: $\frac{3e^{-3t}t^2}{2}$

2. $\frac{2s}{(s+2)^2-5}$ Answer: $2e^{-2t}\cosh\sqrt{5}t-\frac{4}{\sqrt{5}}e^{-2t}\sinh\sqrt{5}t$

3. $\frac{s+3}{s^2-6s+13}$ Answer: $e^{3t}\cos 2t+3e^{3t}\sin 2t$

4. $\frac{1}{s(s+2)^3}$ Answer: $\frac{1}{2}\left[\frac{-e^{-2t}}{2}\left(t^2+t+\frac{1}{2}\right)+\frac{1}{4}\right]$

5. $\frac{s}{\left(s^2-a^2\right)^2}$ Answer: $\frac{t}{2a}\sinh at$

6. $\frac{s+1}{\left(s^2+2s-8\right)^2}$ Answer: $\frac{e^{-t}\,t\sinh 3t}{6}$

7. $\log\left[\frac{s(s^2+1)}{(s^2-9)}\right]$ Answer: $-\frac{1}{t}[1+2\cos t-2\cosh 3t]$

8. $\log\left[1+\frac{a}{s}\right]$ Answer: $\frac{1}{t}\left[1-e^{-at}\right]$

Inverse Laplace Transforms Using Partial Fraction

Partial Fraction Expansions

1. $\frac{f(s)}{(s+a)(s+b)}=\frac{A}{s+a}+\frac{B}{s+b}$ where f(s) is linear

2. $\frac{f(s)}{(s+a)(s+b)(s+d)}=\frac{A}{s+a}+\frac{B}{s+b}+\frac{C}{s+d}$ where f(s) is Quadratic or linear

3. $\frac{f(s)}{(s+a)^2(s+b)}=\frac{A}{s+a}+\frac{B}{(s+a)^2}+\frac{C}{s+b}$ where f(s) is linear or Quadratic

4. $\frac{f(s)}{\left(as^2+bs+c\right)(s+d)}=\frac{As+B}{as^2+bs+c}+\frac{C}{s+d}$ where f(s) is linear or Quadratic and

   as^2+bs+c is non factorisable

5. $\frac{f(s)}{\left(as^2+bs+c\right)(s+d)^2}=\frac{As+B}{as^2+bs+c}+\frac{C}{s+d}+\frac{D}{(s+d)^2}$

   where f(s) is of degree atmost 3and as^2+bs+c is non factorisable

6. $\frac{f(s)}{\left(s^2+a^2\right)\left(s^2+b^2\right)}=\frac{As+B}{s^2+a^2}+\frac{Cs+d}{s^2+b^2}$

Find the Inverse Laplace Transforms by Partial Fraction method for the following

1. $\dfrac{s}{(s+2)(s-3)}$

2. $\dfrac{s^2+s-2}{(s+3)^2(s-2)}$

3. $\dfrac{3s}{(s+1)^2\left(s^2+4\right)}$

4. $\dfrac{1}{s^3-8}$

5. $\dfrac{2s^3+4s^2-s+1}{s^2\left(s^2-s+2\right)}$

Practice Problems for Partial Fraction Methods
Find the Inverse Laplace Transforms for the following

1. $\dfrac{5s^2-15s-11}{(s+1)(s-2)^2}$ Ans : $e^{-t}+4e^{2t}-7te^{2t}$

2. $\dfrac{s+4}{s(s-1)\left(s^2+4\right)}$ Ans : $-1+e^{t}-\dfrac{1}{2}\sin 2t$

Convolution of two Functions

The convolution of two functions f(t) and g(t) is defined as

$$f(t)*g(t)=\int_0^t f(u)\, g(t-u)\, du$$

Convolution Theorem

If $L[f(t)]=F(s)$ and If $L[g(t)]=G(s)$

Then $L[f(t)*g(t)]=F(s)G(s)$

From Convolution Theorem $L^{-1}[F(s)\cdot G(s)]=L^{-1}[F(s)]*L^{-1}[G(s)]$

Find Inverse Laplace Transforms using Convolution theorem for the following

1. $\frac{1}{s\ (s+1)}$

2. $\frac{1}{s\left(s^2+a^2\right)}$

3. $\frac{1}{(s+a)(s+b)}$

4. $\frac{1}{s^2\ (s+1)}$

5. $\frac{1}{(s+1)\left(s^2+1\right)}$

6. $\frac{1}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$

7. $\frac{s}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$

8. $\frac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$

9. $\frac{s}{\left(s^2+a^2\right)^2}$

Practice Problems for Convolution method

1. $\frac{1}{(s+2)(s+3)}$ Answer: $e^{-2t}-e^{-3t}$

2. $\frac{1}{\left(s^2+a^2\right)^2}$ Answer: $\frac{1}{2a^3}(\text{sinat - at cosat})$

3. $\frac{s^2}{\left(s^2+a^2\right)^2}$ Answer: $\frac{1}{2a}(\text{sinat} + \text{at cosat})$