SATHYABAMA UNIVERSITY

DEPARTMENT OF MATHEMATICS

Engineering Mathematics-IV (SMTX1010)

Question Bank

UNIT I
PART A

1. What is the value of b_n when the function $f(x)=x^2$ expanded as a Fourier series in $(-\pi, \pi)$?
2. State Dirichlet conditions for a function to be expanded as a Fourier series.
3. If $f(x) = \sin x$ in $-\pi \le x \le \pi$, find a_n
4. Write the complex form of Fourier series of f(x) in $0<x<l$.
5. Find the Fourier cosine series for the function $f(x)=k$, $0<x<\pi$
6. State parseval's identity on Fourier series
7. If $f(x) = \sum_{-\infty}^{\infty} c_n e^{inx}$ is the complex form of the Fourier series corresponding to f(x) in $(0,2\pi)$, Write the formula for C_n.
8. Write the formulas for finding the Euler's constants in the Fourier series expansion of f(x) in $(-\pi, \pi)$.
9. Find the root mean square value of the function $f(x) = x$ in the interval (0,1).
10. Define Harmonic analysis.

PART B

1. Obtain the Fourier Series for $f(x) = x + x^2$ in $(-\pi, \pi)$. Deduce that

   $$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+..........\infty=\frac{\pi^2}{6}.$$

2. Expand the function $f(x) = x\sin x$ as a Fourier series in the interval $0 \le x \le 2\pi$.
3. Express $f(x) = x$ in half range cosine series in the range $0 < x < 1$ and deduce the value of $\left(\frac{1}{1^4}\right)+\left(\frac{1}{3^4}\right)+\left(\frac{1}{5^4}\right)+.....to\ \infty$
4. Obtain the Fourier series of $f(x) = |\cos x|$ in $-\pi \le x \le \pi$
5. (a) Find the complex form of Fourier series for $f(x) = e^{-ax}$ in the interval $-1 \le x \le 1$.

(b) Expand $f(x) = 2x - x^2$, $0 < x < 3$ as a half range sine series.

6. Find the Fourier series expansion of the periodic function f(x) of period 2, defined by

$f(x)=\begin{cases} 1+x, & -1 < x \le 0 \\ 1-x, & 0 \le x \le 1 \end{cases}$ Deduce that $\sum_{1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

7. Obtain Fourier series for $f(x)=\begin{cases} l-x, & 0 < x \le l \\ 0, & l \le x \le 2l \end{cases}$. Hence deduce that

$(i)\ 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ... = \frac{\pi}{8}$ $\quad (ii)\ \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...\infty = \frac{\pi^2}{8}$

8. Compute the first two harmonics of the Fourier series of f(x) given by the following table.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	π
f(x)	10.	1.4	1.9	1.7	1.5	1.2	1.0

9. Find the Fourier series of $f(x) = x^2$ in $-\pi \le x \le \pi$ and deduce $\sum_{1}^{\infty} \frac{1}{n^2} = \pi^2/6$

10. Obtain the first two harmonics in the Fourier series expansion in (0,6) for the function $y = f(x)$ defined by the table given below.

X	0	1	2	3	4	5
Y	9	18	24	28	26	20

UNIT II
PART A

1. Form the partial differential equation from $z = (x-a)^2 + (y-b)^2 + 1$ by eliminating a and b.
2. Find the partial differential equation by eliminating arbitrary constants a & b . from z = (x+a)(y+b)
3. Write the complete integral for the partial differential equation z = px +qy +pq .
4. Find the complete solution of $z = px + qy + p^2q^2$
5. Form the p.d.e by eliminating the arbitrary function from $z = f(x^2 + y^2)$.
6. Find the particular integral of $(D^2 - 6DD' + 9D'^2)z = \cos(3x+y)$.
7. Solve $(D^4 - D'^4)z = 0$.
8. Find the particular integral of $(D^2 - DD')z = \sin x \cos 2y$.
9. Find the complete integral of p + q = pq.
10. Solve $(D^3 - 3DD'^2 + 2D'^3)z = 0$.

PART B

1. Find the singular integral of the PDE $z = px + qy + p^2 - q^2$.

2. Form the partial differential equation by eliminating the arbitrary functions 'f' and 'g' in $z = f(2x + y) + g(3x-y)$

3. Solve $(D^2 + DD' - 2D'^2)z = e^{2x+y} + \sin(x+y)$.

4. Solve : $(x^2-yz)p + (y^2 - zx)q = z^2 - xy$.

5. Solve : $x(y^2 + z^2)p + y(z^2 + x^2)q = z(y^2 - x^2)$.

6. Solve $(D^2 - 6DD' + 5D'^2)z = e^x \sinh y$.

7. Solve $(mz - ny)p + (nx - lz)q = ly - mx$.

8. Solve $(D^2 + 4DD' - 5D'^2)z = x + y^2 + \pi$.

9. Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \cos x$.

10. Find the general solution of $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$

UNIT III
PART A

1. The steady state temperature distribution is considered in a square plate with sides x = 0, y = 0, x = a and y = a. The edge y = 0 is kept at a constant temperature T and the other three edges are insulated. The same state is continued subsequently. Express the problem mathematically.
2. State any two laws which are assumed to derive one dimensional heat equation
3. State two-dimensional Laplace equation.
4. How many boundary conditions are required to solve completely $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$
5. What is meant by Steady state condition in heat flow?
6. An insulated rod of length 1cm has its end A and B maintained at 0^oC and 100^oC respectively and the rod is in steady state condition. Find the temperature at any point in terms of its distance x from one end.

7. A bar 20 cm long with insulated sides has its ends kept at 40°C and 100°C until steady state conditions prevail. Find the initial temperature distribution in the bar.
8. A tightly stretched string with fixed end points x = 0 and x = l is initially in a position given by y(x,0) = f(x). If the string is released from rest, write down the most general solution of the vibrating motion of the string.
9. A bar 10 cm long with insulated sides has its ends kept at 30°C and 60°C until steady state conditions prevail. Write the initial and boundary conditions of one dimensional heat equation.
10. Write down various solutions of the one dimensional wave equation.

PART B

1. A taut string of length L is fastened at both ends. The midpoint of the string is taken to a height of b and then released from rest in this position. Find the displacement of the string at any time t.
2. A tightly stretched string with fixed end points x = 0 and x = l is initially in a position given by y (x, 0) = $V_o \sin^3(\pi x/l)$. If it is released from rest from this position. Find the displacement y at any distance x from one end at any time t.

3. A string AB of length 'l'tightly stretched between A and B is initially at rest in the equilibrium position. If each point of the string is suddenly given a velocity $\lambda(lx - x^2)$ find the transverse displacement of the string at subsequent time t.

4. If a string of length l is initially at rest in equilibrium position and each point of it is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{(x,0)} = V_0 \sin^3\left(\frac{\pi x}{l}\right), 0 < x < l.$ determine the transverse displacement y(x,t).
5. The end A and B of a bar of length l maintained at 0^oC and 100^oC respectively until steady state conditions prevail. If the temperature at B is reduced suddenly to 0^oC and kept so. Find the temperature u(x, t) at a distance x from A and at time t.
6. The two long edges y = 0 and y = l of a long rectangular plate are insulated. If the temperature in the short edge at infinity is kept at 0^oC, while that in the short edge x = 0 is kept at ky(l – y), find the steady-state temperature distribution in the plate.

7. A rod of length 30 cms has its ends A and B kept at 20°C and 80°C respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature function u(x,t).

8. A square plate bounded by the lines x=10, y=10,y=0 are kept at constant temperature 0° c whereas the next edge x=0 is kept at 100˚c. Find the distribution over the plate.

9. A rod of length 30 cms has its ends A and B kept at 20°C and 80°C respectively until steady state conditions prevail. The temperature at end A is then suddenly increased to 60°C and at end B decreased to 60°C . Find the subsequent temperature function u(x,t).

10. Find the Steady state temperature at any point of a square plate whose two adjacent edges are kept at 0°C and and the other two edges are kept at the constant temperature 100°C

UNIT IV
PART A

1. State Parseval's identity for Fourier transforms
2. Find finite fourier sine transform of f(x) = K in (0,2)
3. If F(s) is the Fourier transform of f(x), write the Fourier transform of f(x)sin *ax* in terms of F.T
4. If F(s) = F(f(x)) then show that F(f'(x)) = -is F(s).
5. Find the Fourier sine transform of e^{-ax}.
6. State the shifting property on Fourier transforms.
7. Find the Fourier transform of f(x) if
$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$$
8. Prove that $F\{f(ax)\} = \frac{1}{a} F\left[\frac{s}{a}\right]$
9. Prove that $Fs[xf(x)] = -\frac{d}{ds} Fc[f(x)]$
10. State Convolution Theorem.

PART B

1) Find the Fourier transform of $f(x) = \begin{cases} 1-|x|, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ and

hence deduce that $(i)\ \int_0^\infty \left(\frac{\sin t}{t}\right)^2 dx \qquad (ii)\ \int_0^\infty \left(\frac{\sin t}{t}\right)^4 dx$

2) Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & |x| \le a \\ 0, & |x| > a \end{cases}$ and Hence deduce that

$(i)\ \int_0^\infty \frac{\sin t - t\cos t}{t^3} dt \qquad (ii)\ \int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt$

3) Find the Fourier sine transform of e^{-ax} and hence evaluate $\int_0^\infty \frac{x \sin sx}{1+x^2} dx$

4) Find the Fourier cosine transform of $e^{-a^2x^2}$ and hence evaluate the fourier sine transform of $xe^{-a^2x^2}$.

5) Using Parseval's identity on Fourier transform evaluate $\int_0^\infty \frac{dx}{\left(x^2+a^2\right)^2}$ & $\int_0^\infty \frac{x^2 dx}{\left(x^2+a^2\right)^2}$

6) Find the Fourier transform of $e^{-a^2x^2}, a>0$.

For what value of 'a' the given function is self reciprocal

and hence show that $F\left[e^{-\frac{x^2}{2}}\right] = e^{-\frac{s^2}{2}}$

7) Find $F_c\left[\frac{1}{x^2+a^2}\right]$ and $F_s\left[\frac{x}{x^2+a^2}\right]$

8) State and prove Parseval's identity on Fourier transform .
9) State the prove convolution theorem on Fourier transform .
10) Evaluate $\int_0^\infty \frac{dx}{\left(x^2+a^2\right)\left(x^2+b^2\right)}$ using Fourier cosine transforms

UNIT V
PART A

1. If F (z) = $\frac{z^2}{(z-\frac{1}{2})(z-\frac{1}{4})(z-\frac{3}{4})}$, find f (0).
2. Find the Z-transform of x(n) = $\begin{cases} \frac{a^n}{n!} & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$
3. Define Z-Transforms.
4. Find Z[$a^n n$].
5. Find the Z- transform of $na^n u(n)$.
6. Find Z[e^{at+b}].

7. Find $Z[a^{n-1}]$.
8. Prove that $Z[f(n+1)] = z[F(z) - f(0)]$
9. Prove that $z[nf(n)] = -z\dfrac{dF(z)}{dz}$
10. Define Convolution of two sequences.

PART B

1. Solve the equation $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$ given $u_0 = u_1 = 0$
2. Using convolution theorem find $Z^{-1}\left[\dfrac{z^2}{(z-4)(z-3)}\right]$
3. Find the inverse Z-transform of $\dfrac{z^3 - 20z}{(z-2)^3(z-4)}$
4. State and prove convolution theorem on Z-transforms.
5. Find $Z^{-1}\left[\dfrac{9z^3}{(3z-1)^2(z-2)}\right]$ by using residue method.
6. Find $Z[a^n r^n \cos n\theta]$ and $Z[a^n r^n \sin n\theta]$
7. Prove that $z\left[\dfrac{1}{(n+1)}\right] = z\log\left[\dfrac{z}{z-1}\right]$
8. Find $z^{-1}\left[\dfrac{2z^2 - 10z + 13}{(z-3)^2(z-2)}\right]$ when $2 < |z| < 3$
9. Using Z-transform solve the equation $y_{n+2} + 4y_{n+1} - 5y_n = 24n - 8$, given y(0)=3 and y(1) = -5.
10. Find the inverse Z-transform of $\dfrac{10z}{z^2 - 3z + 2}$